Geometry of Moduli Space of Low Dimensional Manifolds

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Monday, December 14

On the arc metric on Teichmüller space

Athanase Papadopoulos
University of Strasbourg

Abstract: The arc metric is an asymmetric metric on the Teichmüller space of a surface with boundary, which is the analogue of Thurston's metric on the Techmüller space of surfaces without boundary. I will present several properties of this metric including results on its geodesics and on its boundary structure.

Index theory on Lorentzian manifolds

Christian Bär
University of Potsdam

Abstract: Starting with the classical Gauss-Bonnet theorem we give a short historical introduction to index theory for elliptic operators on Riemannian manifolds. Then we show that the Dirac operator on a compact globally hyperbolic Lorentzian spacetime with spacelike Cauchy boundary is a Fredholm operator if appropriate boundary conditions are imposed. We prove that the index of this operator is given formally by the same expression as in the index formula of Atiyah-Patodi-Singer for Riemannian manifolds with boundary.

This is the first index theorem for Lorentzian manifolds. We explain the methods which enter the proof; they are, from an analytic perspective, quite different from the classical Riemannian case. Finally, we discuss an application in quantum field theory. This is joint work with Alexander Strohmaier.

Tuesday, December 15

Dynamics and geometry of fibered 3-manifolds and of free-by-cyclic groups.

Ilya Kapovich
University of Illinois at Urbana-Champaign

Abstract: The "fibered face" theory of Thurston describes, in algebraic, geometric and topological terms, how a given 3-manifold M can fiber over the circle in multiple ways. The answer is given by the "Thurston norm" on $H^1(M, R)$. The unit ball with respect to this norm is a compact rational centrally symmetric polyhedron, with a distinguished collection of top-dimensional open faces called "fibered faces". A primitive integral cohomology class $u \in H^1(M, R)$ corresponds to a fibration of M over the circle if and only if u lies in a positive cone over some fibered face. Later, Fried and McMullen refined Thurston's theory and described, in a uniform way, the dynamics corresponding to different ways in which M fibers over the circle. In particular, Mcmullen's "Teichmuller polynomial" captures the information about the stretch factors of the monodromies of all such fibrations. After reviewing the 3-manifold case, we will discuss its algebraic counterpart given by free-by-cyclic groups, that is, mapping tori of automorphisms of finite rank free groups. We will describe a version of "fibered face" theory in the free-by-cyclic context, including a (partial) replacement for the Thurston norm, and the notion of a "McMullen polynomial" capturing the information about stretch factors of monodromies of different splittings of a given group as a free-by-cyclic group. We will also discuss the relationship of the 3-manifold and free-by-cyclic settings with the BNS invariant for finitely presented groups. The talk is based on several joint papers with Spencer Dowdall and Christopher Leininger.

Conjugation classes of pairs in $SL(4, \mathbb{C})$ and $SU(3, 1)$

Krishnendu Gongopadhyay
Indian Institute of Science Education and Research (IISER) Mohali

Abstract: We first describe a symmetric minimal global coordinate system on the $SL(4, \mathbb{C})$-character variety of a rank 2 free group F_2. Restricting this coordinate system gives a smaller global trace coordinates on the representation variety of F_2 into $SU(3, 1)$. This is a joint work with Sean Lawton.

Estimates of eigenvalues of Laplacian by a reduced number of subsets

Kei Funano
Kyoto University

Abstract: Chung-Grigory'an-Yau's inequality describes upper bounds of eigenvalues of Laplacian in terms of subsets ("input") and their volumes. In this talk I will explain how to reduce "input" in Chung-Grigory'an-Yau's inequality in the setting of Alexandrov spaces satisfying $\mathrm{CD}(0, \infty)$. I will also discuss a related conjecture for some universal inequality among eigenvalues of Laplacian.

Wednesday, December 16

Compactifications of Margulis space-times

Suhyoung Choi
Korea Advanced Institute of Science and Technology

Abstract: Let $\mathbf{R}^{2,1}$ be a complete flat Lorentzian space of dimension 3, and let Γ be a freely and properly acting Lorentzian isometry group isomorphic to a free group of rank $r \geq 2$. The quotient space $\mathbf{R}^{2,1}/\Gamma$ is an open 3-manifold, called a *Margulis space-time*, as first constructed by Margulis and Drumm in 1990s. We will talk about the compactification of Margulis space-times by attaching closed $\mathbf{R}P^2$-surfaces at infinity when the groups do not contain parabolics. The compactified spaces are homeomorphic to solid handlebodies. Finally, we will discuss about the parabolic regions of tame Margulis space-times with parabolic holonomies. This is a joint work with William Goldman. (There is an another approach by Danciger, Kassel, and Gueritaud.)

Dynamical systems arising from mapping class group actions on moduli spaces

William Goldman
University of Maryland

Abstract: The action of the mapping class group action on character varieties (that is, moduli spaces of representations of fundamental groups of surfaces) is a fundamental group action. It arises from the classification of geometric structures on manifolds, gauge theory, and provides a rich supply of topological invariants.

My frist talk will survey this theory. My second talk will dicsuss the case of the rank two free group and PGL(2,R)-representations. This is the theory pioneered pioneered by Brian Bowditch in his paper "Markoff triples and quasi-Fuchsian groups". I will discuss joint work with Ser-Peow Tan, George Stantchev and Greg McShane.

Thursday, December 17

Flat surfaces: rigidity, blocking, illumination

Samuel Lelievre
University of Paris-Sud (Paris 11)

Abstract: To be announced.

Representation varieties detect essential surfaces

Takahiro Kitayama
Tokyo Institute of Technology

Abstract: Culler and Shalen established a method to construct essential surfaces in a 3-manifold from an ideal point of the SL_2-character variety of the 3-manifold group. Hara and I presented an analogous extension of the method to the case of higher dimensional representations. In general, a certain kind of branched surface (possibly without any branch) is given by an ideal point of the SL_n-character variety, and corresponds to a nontrivial splitting of the 3-manifold group as a complex of groups. The construction is based upon the interplay among the geometry of representation varieties, the theory of Bruhat-Tits buildings, and the topology of 3-manifolds. Essential surfaces in some 3-manifolds known to be not detected in the classical SL_2-theory, and a question is whether any of such essential surfaces is detected in the SL_n-theory or not. We show that every essential surface in a 3-manifold is given by an ideal point of the SL_n-character variety for some n. The talk is partially based on joint works with Stefan Friedl and Matthias Nagel, and also with Takashi Hara.

The Pressure metric for Anosov representations

Martin Bridgeman
Boston College

Abstract: Using thermodynamic formalism, we introduce an intersection for convex representations. Taking the associated Hessian metric, we produce an Out-invariant Riemannian metric on the smooth points of the deformation space of projective, irreducible representations of a word hyperbolic group G into SL(m, R) whose Zariski closure contains a generic element. In particular, we produce a mapping class group invariant Riemannian metric on Hitchin components which restricts to the Weil-Petersson metric on the Fuchsian locus. This is joint work with R. Canary, F. Labourie and A. Sambarino.

Friday, December 18

Harmonic maps of Riemann surfaces into symmetric spaces and their moduli spaces

Yoshihiro Ohnita
Osaka City University

Abstract: The purpose of my two talks is to provide a survey on harmonic map theory of Riemann surfaces into symmertric spaces and their moduli spaces. In Part I, I will begin with the zero curvature representation of the harmonic map equations and the loop group formulation in infinite dimensional Grassmannian models. As the most fundamental structures of such harmonic maps, I will explain the loop group actions (Uhlenbeck, Guest-O.) and the Weierstrass type represention formula (Dorfmeister-Pedit-Wu) for harmonic maps. In Part II, I will discuss the gauge-theoretic formulation of harmonic maps and the moduli spaces of solutions to the gauge-theoretic equations (Hitchin, Mariko Mukai-Hidano, etc.). I will also mention about recent related results.

Moebius structures on the boundary of hyperbolic spaces

Viktor Schroeder
University of Münich

Abstract: A main feature of the classical hyperbolic space $\mathbb{H}^n$ is the deep relation between the geometry of this space and the Möbius geometry of its boundary at infinity. For example the isometries of the hyperbolic space correspond to Möbius transformations of its boundary. Many of these relations can be generalized to Riemannian manifolds of negative curvature and even more general to so called CAT(-1) spaces or even to Gromov hyperbolic spaces. In the lectures we explain what a Möbius structure is and how one how can associate to generalized hyperbolic spaces a Möbius structure at infinity. Then we study the (much more complicated) inverse problem: to what extend can one reconstruct the hyperbolic space from the Möbius structure. In particular we want to study this problem for Möbius structures on S^1 (which correspond to the boundary at infinity of surfaces). This question is related to many classical problems: e.g. problems concerning the conjugacy problem of the geodesic flow on surfaces and questions the marked length spectrum. In general the question is very open and only partial answers are known.