Geometry and Analysis

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The general positive mass theorem I, II III, IV

Richard Schoen
University of California, Irvine

Abstract:

Abstract: We will discuss the natural extensions of the positive mass theorem to higher dimensions. The approaches which have been successful in high dimensions are the minimal hypersurface approach and the Dirac operator approach. Both of these only work directly under additional assumptions. We will describe the minimal hypersurface approach and how it can be extended across singularities which may occur in sufficiently high dimensions.

On birth and spread type nonlinear partial differential equations

Hiroyoshi Mitake
University of Tokyo

Abstract: In this talk, we introduce birth-spread type nonlinear partial differential equations which is motivated by a crystal growth phenomenon. Mathematically, an interesting nonlinear phenomenon in terms of asymptotic speed of solutions appears which is sensitive to the shapes of source terms.We discuss properties of large-time asymptotic speed, and also present recent results of large-time convergence of solutions. This is a joint work with Y. Giga (Univ. of Tokyo) and H. V. Tran (Univ. of Wisconsin-Madison).

Attainability of the best Sobolev constant in a ball

Norisuke Ioku
Tohoku University

Abstract: The best constant in the Sobolev inequality in the whole space is attained by the Aubin–Talenti function; however, this does not happen in bounded domains because of the break down of the dilation invariance. In this talk, we investigate a new scale invariant form of the Sobolev inequality in a ball and show that its best constant is attained by functions of the Aubin–Talenti type. Some relationship between our result in a ball and the Sobolev inequality in the whole space is also discussed.

References

[1] N. Ioku, Attainability of the best Sobolev constant in a ball, Math. Annalen **375** (2019), 1–16.

Surface groups acting on CAT(−1) spaces

Chikako Mese
Johns Hopkins University

Abstract: The relationship between dynamical properties of discrete group actions on metric spaces and rigidity theorems has a rich history. A result due to Bowen states that the Hausdorff dimension of the limit set Λ of a quasi-Fuchsian group Γ acting on hyperbolic 3-space $\mathbb{H}^3$ is equal to 1 if and only if Γ is Fuchsian. A generalization of this result to surface group actions on CAT(−1) metric spaces was originally conjectured by Bourdon and was later verified by Bonk and Kleiner where they prove a more general statement about quasi-convex actions.

In this talk, we will discuss a harmonic maps approach to this problem and give a new proof of the original conjecture of Bourdon. The main result is the following: *Given a convex cocompact action* $\rho : \pi_1(S) \to Isom(X)$ *on a* $CAT(-1)$ *metric space* X *by the fundamental group of a closed, connected oriented surface* S *with genus* > 1 *such that* $\dim_{\mathcal{H}}(\Lambda) = 1$, *there exists a hyperbolic metric* h *on the universal cover* $\tilde{S}$ *of* S *such that the unique* ρ*-equivariant harmonic map* $\tilde{u} : \mathbb{H}^3 = (\tilde{S}, h) \to X$ *is a totally geodesic map and an isometric embedding.* This is a joint work with G. Daskalopoulos, A. Sanders and A. Vdovina

Riemannian metrics maximizing the first eigenvalue of the Laplacian on a closed surface

Shin Nayatani
Nagoya University

Abstract: Abstract: By the classical work of Hersch-Yang-Yau, the first eigenvalue of the Laplacian (under area normalization) on a closed surface has an explicit upper bound in terms of the genus of the surface. In this talk, I will focus on Riemannian metrics that maximize the invariant. I overview the recent progress on the existence problem for maximizing metrics, including the affirmative resolution of Jacobson-Levitin-Nadirashvili-Nigam-Polterovich's conjecture, which explicitly predicts maximizing metrics in the case of genus two, by Toshihiro Shoda and myself.