Working Workshop on
Calabi-Yau Varieties and Related Topics 2023

Jul.22 (Sat.),23 (Sun.), 2023

Science Build. #4, room 501, Hokkaido University

Jul.22 (Sat.)

10:30–11:30 Ichiro Shimada (Hiroshima Univ.)

Conway theory for algebraic geometers

13:00–14:00 Yuta Takada (Hokkaido Univ.)

Entropy of K3 surface automorphisms: Lattice theoretic approach

14:30–15:30 Taizan Watari (Kavli IPMU)

On the Gukov-Vafa Conjecture

16:00–17:00 Masato Kuwata (Chuo Univ.)

Toward the theory of Mordell-Weil lattices of elliptic threefolds

Jul.23 (Sun.)

9:30–10:30 Makoto Miura (Osaka Univ.)

Geometric transitions for Calabi-Yau hypersurfaces

10:45–11:45 Ryo Negishi (Hokkaido Univ.)

Monomial deformations of Fermat hypersurfaces and Picard-Fuchs equations

12:00–13:00 Masanori Asakura (Hokkaido Univ.)

Frobenius structure on hypergeometric differential equations and p-adic polygamma functions

Orgainzers: M. Asakura (Hokkaido Univ.), Y. Goto (Hokkaido Edu. Univ.), S. Hosono (Gakushuin Univ.), N. Yui (Queen's Univ.)

Title and Abstracts

Jul.22

Ichiro Shimada (Hiroshima Univ.)

Title: Conway theory for algebraic geometers

Abstract: I present an introductory account on Conway's classical theory about the Leech lattice in terms of geometry of $K3$ surfaces, and give some applications to the calculation of automorphism groups of $K3$ surfaces and Enriques surfaces.

Yuta Takada (Hokkaido Univ.)

Title: Entropy of K3 surface automorphisms: Lattice theoretic approach

Abstract: For any complex $K3$ surface X, its second cohomology group $H^2(X, \mathbb{Z})$ with the intersection form is an even unimodular lattice of signature $(3, 19)$. Such a lattice is unique up to isomorphism and called a $K3$ lattice. The entropy of an automorphism f of a $K3$ surface X is given by the logarithm of the spectral radius of the induced homomorphism $f^* : H^2(X) \to H^2(X)$. On the other hand, it is known that any isometry of a $K3$ lattice preserving some additional properties can be seen as the induced homomorphism of an automorphism of some $K3$ surface. In this talk, I explain a criterion for a given polynomial to be realized as the characteristic polynomial of an isometry of an even unimodular lattice. As an application, the logarithm of every Salem number of degree 20 is realizable as the entropy of an automorphism of a non-projective $K3$ surface.

Taizan Watari (Kavli IPMU)

Title: On the Gukov--Vafa Conjecture

Abstract: A superconformal field theory (SCFT) is assigned to a Ricci flat Kahler manifold (M, ω, J); that is what string theory does. It is known, when (M, ω, J) is an elliptic curve, that the SCFT is a rational CFT if and only if (M, J) and its mirror both have complex multiplications. Gukov–Vafa conjectured in 2002 that this characterization of rational SCFTs in terms of CM-type Hodge structure is more general than just in the case of elliptic curves. We examine the case of abelian surfaces, refine the statements of the conjecture, and establish the characterization. We will also discuss prospects as well as open problems for the case of $K3$ and higher dimensional Ricci flat Kahler manifolds. This presentation is based on arXiv:2205.10299, arXiv:2212.13028 and arXiv:2306.xxxxx in collaboration with Abhiram Kidambi and Masaki Okada.

Masato Kuwata (Chuo)

Title: Toward the theory of Mordell-Weil lattices of elliptic threefolds

Abstract: Mordell-Weil lattice of an elliptic surface is the quotient of its Mordell-Weil group by the torsion subgroup equipped with the NÃľron-Tate pairing. It is a very powerful tool at the crossroads of algebraic geometry and number theory. It is natural to ask whether we can extend the theory to the case of elliptic n-folds. Since there is a symmetric bilinear pairing on the Picard group of an elliptic n-fold, it is plausible to define a height pairing on the Mordell-Weil group. We explain where the difficulties are, and we establish the pairing for a very simple case. We show that even this simple case has an application.

Jul.23

Makoto Miura (Osaka)

Title: Geometric transitions for Calabi--Yau hypersurfaces

Abstract: A geometric transition is an operation connecting two smooth or mildly singular Calabi--Yau varieties by a birational contraction followed by a flat deformation. A famous question, posed by Miles Reid, is whether all smooth Calabi--Yau 3-folds are connected via a sequence of geometric transitions. As an approach to this problem, Mark Gross introduced the idea of using an analogy of the minimal model program for geometric transitions. By refining his idea, one can show that all elliptic curves which are elephants of projective surfaces are connected via smooth transitions associated with blow-ups and blow-downs of ambient surfaces. In this talk, I plan to explain this result, introduce a few examples of geometric transitions, and discuss the way to generalize it to higher dimensions.

Ryo Negishi (Hokkaido)

Title: Monomial deformations of Fermat hypersurfaces and Picard-Fuchs equations

Abstract: The Picard-Fuchs equation is the D-module which corresponds to the local system Rf_*C of a smooth proper morphism f. The relative de Rham cohomology of the monomial deformation of a Fermat hypersurface is decomposed by an action of some finite abelian group. In this talk, I show that the Picard-Fuchs equation of each component is a certain generalized hypergeometric equation. As an application, I compute Katz' deformation matrix of the above family that is obtained by Kloosterman in a different manner.

Masanori Asakura (Hokkaido)

Titile: Frobenius structure on hypergeometric differential equations and \$p\$-adic polygamma functions

Abstract: Kedlaya constructs the Frobenius structure on hypergeometric equations using GKZ(=Gelfand, Kapranov, Zelevinsky) equations. The explicit description of the Frobenius matrix is given as series expansions of rigid analytic functions, and he provides the formula on the residue at 0 in terms of certain products of p-adic gamma functions. However, some cases are missing in his result, e.g. $F(a_1, ..., a_n; 1, ..., 1; z)$. In this talk, we fill in it. In particular, we show that the residue at 0 in the missing cases are described by the p-adic polygamma functions. This is a joint work with Kei Hagihara.