Summary

We discuss dynamics of a particle on a metric quantum graph Γ as limit when $\epsilon \to 0$ of the evolution given by the Schroedinger equation with a strong constraining potential

$$i\hbar\frac{\partial\phi}{\partial t} = -\frac{1}{2}\Delta\phi + V(x) + \frac{1}{2\epsilon^2}W(x)$$

where $V(x)$ is smooth and $W(x)$ is smooth, zero on Γ and strictly positive elsewhere.
A similar analysis can be done if the constraining potential is subsituted with Dirichlet boundary conditions at the boundary of a tube of radial size ϵ surrounding the graph.
A concrete example of set-up is given by carbon nanotubes and also by the motion of valence electrons in aromatic molecules or periodic solids like graphene.
We prove that the limit function that one obtains projecting (in a suitable sense) the solution of the equation for $\epsilon > 0$ on the graph satisfies a natural limit Schroedinger in the open domain Γ_0 obtained from the graph by deleting the vertices; the dynamics is then generated by a self-adjoint extension of the Schroedinger operator restricted to Γ_0 (point interactions).
The resulting extension depends on the detailed form of the constraining potential (or the form of the tubular neigborhhod in a neighborhood of the vertices). We shall give examples in which different limit dynamics are obtained.
We will discuss open problems, and results (and open problems) that refer to the semi-classical limit.