The Fermi Golden Rule at Thresholds

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Abstract

Let $H = -\Delta + V$ on $L^2(\mathbf{R}^3)$. Assume $V \in C_0^\infty(\mathbf{R}^3)$. Assume furthermore that 0 is a non-degerenrate eigenvalue of H with normalized eigenfunction Ψ_0. Let $W \in C_0^\infty(\mathbf{R}^3)$, and assume $W(x) \geq 0$. Consider the family $H(\varepsilon) = H + \varepsilon W$ with $\varepsilon > 0$. What happens to the eigenvalue under this perturbation, for ε sufficiently small? We show that it turns into a resonance. Its location is given by a modified Fermi Golden Rule. Let $\lambda(\varepsilon) = x_0(\varepsilon) - i\Gamma(\varepsilon)$ be the location of the resonance. Then under some assumptions we find that there is an odd integer ν, $\nu \geq -1$, such that

$$\begin{aligned} x_0(\varepsilon) &= b\varepsilon(1 + \mathcal{O}(\varepsilon)), \\ \Gamma(\varepsilon) &= -i^{\nu-1} g_\nu b^{\nu/2} \varepsilon^{2+(\nu/2)}(1 + \mathcal{O}(\varepsilon)), \end{aligned}$$

where $b = \langle \Psi_0, W\Psi_0 \rangle$. The coefficient g_ν is computed explicitly in terms of W, Ψ_0, and the coefficients in the asymptotic expansion of $(H - z)^{-1}$ around zero.

The main results are obtained in an abstract framework, modelled on the above example of an application of our main theorem.

The results presented are joint work with G. Nenciu, Bucharest.