Recently Naoto Kumano-go (Bulletin des Sciences Mathématiques,vol.128 (2004)) succeeded in proving that piecewise linear time slicing approximation to Feynman path integral with integrand $F(\gamma)$ actually converges to the limit if the functional $F(\gamma)$ of paths γ belongs to a certain class of functionals, which includes, as a typical example, Stieltjes integral of the following form;

$$F(\gamma) = \int_0^T f(t, \gamma(t))\rho(dt), \tag{1}$$

here $\rho(t)$ is a function of bounded variation and $f(t, x)$ is a sufficiently smooth function with polynomial growth at $|x| \to \infty$.

We shall write down the second term of the semi-classical asymptotic series of the Feynman path integrals if the integrand is of the form (1). If $F(\gamma) \equiv 1$, this second term coincides with the one given by G.D. Birkhoff in his famous paper (Bull. Amer. Math. Soc. vol 39,(1933). Otherwise, we believe the formula is new as far as we know.