Geometry of Moduli Space of Low Dimensional Manifolds

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Monday, January 7

Understanding circle homeomorphisms using hyperbolic geometry I & II

Dragomir Saric
City University of New York

Abstract: The Universal Teichmüller space $T(D)$ is the space of all quasisymmetric homeomorphisms of the unit circle S^1 that fix three points. Thurston introduced the notion of (left) earthquake maps on the unit disk D using the hyperbolic geometry of D. The trace of an earthquake map on the unit circle S^1 is a homeomorphisms of S^1. Thurston proved that any homeomorphism of S^1 is the trace of a unique earthquake map of D. The induced map is a quasisymmetric homeomorphism if and only if the supremum of the mass of the earthquake measure over all fixed size sets is finite. Thus we obtain a parametrization of $T(D)$ by bounded earthquake measures, i.e. bounded measured laminations.

A related construction of shear maps starts from a fixed tesselation of the unit disk D and a finitely additive signed measure on the tesselation to produce a map of D to D. For some of the signed measures the traces of their shear maps are homeomorphisms of S^1. Unlike in the case of earthquakes, any homeomorphism of S^1 can be obtained from a single tesselation of D by choosing different signed measures. The disadvantage is that the measures are only finitely additive.

Penner posed a problem of deciding when shear measures on the Farey tesselation give rise to homeomorphisms, quasisymmetric maps, symmetric maps, etc. Penner and Sullivan gave a sufficient condition on the signed measure such that the induced map is quasisymmetric. The large group of symmetries of the Farey tessellation allowed us to give a necessary and sufficient condition on the signed measures on the Farey tesselation for the induced maps of S^1 to be homeomorphisms, quasisymmetric and symmetric maps. Thus we obtained a parametrization of $T(D)$ using the shear coordinates on the Farey tesselation. In addition, we characterize the Zygmund vector fields on S^1 (i.e. the tangent vectors to $T(D)$ at the basepoint) in terms of their shear coordinates on the Farey tessellation. The almost complex structure on $T(D)$ is given by Hilbert transform on the Zygmund vector fields. We find a formula for the Hilbert transform in terms of the shear coordinates.

Extendability of conformal structures on punctured surfaces I & II

Jingyi Chen
University of British Columbia

Abstract:

In two-dimensional geometric variational problems it is important to know whether isolated singularities are removable while preserving conformal properties. We shall discuss the problem of extending conformal structure of an immersion from a punctured two-dimensional disk across the puncture as a branched conformal immersion, in two cases: first when the mean curvature satisfies certain integral bound, and second when the immersion has low regularity.

Tuesday, January 8

Introduction to Teichmüller space

Inkang Kim
Korea Institute of Advanced Study

Abstract:
This part is for beginners and graduate students. I will recall the definition of Teichmüller space, metrics over it and some notions for energy of smooth maps between Riemannian manifolds for the second part.

Plurisubharmonicity and geodesic convexity of energy function on Teichmüller space

Inkang Kim
Korea Institute of Advanced Study

Abstract: We describe the first and second derivatives of energy function from a Riemannian manifold into hyperbolic surface depending on the hyperbolic structure, and show plurisubharmonicity on Teichmüller space. We also show the convexity of geodesic length function along Weil-Petersson geodesics. This is a joint work with Zhang and Wan.

Nilpotent groups, Closed geodesics and Heat kernels I & II

Atsushi Katsuda
Kyushu University

Abstract: Geometric analogues of the prime number theorem are established by Selberg, Margulis, Parry-Pollicott etc., which are called the prime geodesic theorems. In this talk, I will explain some refinements, which are geometric analogues of the Chebotarev density theorem, especially for nilpotent extensions. By a common strategy in some parts, we can also obtain an asymptotic expansion for long time behavior of the heat kernels for complete Riemannian manifolds with nilpotent symmetry. Our methods are combinations of the representation theory for discrete groups,the semi-classical analysis and the Chen's iterated integrals.

Wednesday, January 9

The bounds of Hausdorff measure of nodal sets

Jui-En Chang
National Taiwan University

Abstract: Finding an asymptotic bound of the measure of the nodal set is an interesting problem which is first posed by Yau for the Laplacian operator. Recently, the optimal lower bound is established for closed smooth manifolds. In this talk, I'll give a survey on some of the estimation of the Hausdorff measure of nodal sets of several different problems and my work about biharmonic operators.

The uniqueness of self-shrinking networks with 2 closed regions

Jui-En Chang
National Taiwan University

Abstract: The network flow is a generalization of the curve shortening flow from curves to planar networks. To study the formation of singularities, solutions which move by homothety scaling plays an important role. Such network is called a regular shrinker. Up to now, only the shrinker with less than 2 triple junctions or 1 closed region is classified. In this talk, I'll present our recent result about the classification of regular shrinker with 2 closed regions. This is a joint work with Dr. Yang-Kai Lue.

Mapping class groups via group actions I & II

Harry Baik
Korea Advanced Institute of Science and Technology

Abstract: We discuss the action of mapping class groups on various spaces, and study what those actions tell us about the mapping class groups and their subgroups.

Thursday, January 10

Geodesics on hyperbolic surfaces I & II

Yi Huang
Yau Mathematical Sciences Center, Tsinghua University

Abstract: Geodesics (especially simple closed geodesics) are one of the most elementary objects in theory of hyperbolic surfaces. They help decompose complicated surfaces into simpler ones; they help to parameterize Teichmüller spaces; they serve as helpful analogies for understanding certain aspects of higher dimensional hyperbolic manifold theories. And yet, how much do we really understand their behavior?

We focus on two particular results to do with hyperbolic surfaces:

- the Birman-Series theorem on the sparsity of geodesics, and
- McShane identities — a type of infinite term trigonometric identity for hyperbolic surfaces.

We will see how to prove these results, how to generalize them, as well as how they intimately relate with several deep results regarding hyperbolic surfaces.

Harmonic maps and geometric rigidity I & II

Chikako Mese
Johns Hopkins University

Abstract: Harmonic maps from a Riemannian manifolds to NPC spaces (a complete metric space with non-negative curvature in the sense of Alexandrov triangle comparison that include Riemannian manifolds of non-positive sectional curvature) are minimizers of the energy functional. Harmonic map turns out to be a very special maps in many important situations. Indeed, in application of harmonic map theory to rigidity questions, a crucial step is to prove that a harmonic map is pluriharmonic, holomorphic, totally geodesic or even isometric.

In the first talk, we will survey applications of harmonic map theory to geometric rigidity problems including Siu's strong rigidity theorem for Kähler manifolds generalizing a special case of Mostow rigdidity and Corlette and Gromov-Schoen's rank 1 superrigidity theorems complementing Margulis superrigidty. In the second talk, we will discuss a joint work with G. Daskalopoulos on the holomorphic rigidity for Teichmüller space.