Geometry of Moduli Space of Low Dimensional Manifolds

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Tuesday, January 9

A panorama on Riemann Surfaces going from Topology to Algebraic Geometry I & II

Nobert A'Campo
University of Basel

Abstract: Let Γ be a finitely generated group and let $G/\mathbb{Q}$ be a simple algebraic group. It is a dream to understand in this generality the space/variety of all representations $X(\Gamma, G) = \mathrm{Rep}(\Gamma, G)/G$ up to conjugacy. A very general statement due to Grothendieck shows that it only depends on the profinite completion of the group Γ, and on the algebraic group G. For instance, the ultra-product construction of the field of complex numbers $\mathbb{C}$ shows that $X(\Gamma, G)(\mathbb{C})$ is determined by the family $X(\Gamma, G)(F)$ where F runs over all finite fields.

The Mostow Rigidity Theorem asserts that two compact, connected hyperbolic three manifolds are isometric if the fundamental groups are isomorphic. Ian Agol lifts the Mostow Rigidity Theorem to a conjecture: two compact, connected hyperbolic three manifolds are isometric if the profinite completions of the fundamental groups are isomorphic.

The Theory of Riemann Surfaces is a corner stone for the field of investigations that is opened by the above dream. The aim of the lecture is to give a panorama on Riemann Surfaces going from Topology to Algebraic Geometry.

Diffeomorphism groups of intermediate regularity I & II

Thomas Koberda
University of Virginia

Abstract: Let M be the interval or the circle. For each real number $\alpha \in [2, \infty)$, write $\alpha = k + \tau$, where k is the floor function of α. I will discuss a construction of a finitely generated group of diffeomorphisms of M which are C^k and whose k^{th} derivatives are τ–Hölder continuous, but which are admit no algebraic smoothing to any higher Hölder continuity exponent. In particular, such a group cannot be realized as a group of C^{k+1} diffeomorphisms of M. I will discuss the construction of countable simple groups with the same property, and give some applications to continuous groups of diffeomorphisms. This is joint work with Sang-hyun Kim.

Wednesday, January 10

On the equivalence between Bowditch's Q-condition and the primitive stable condition I & II

Binbin Xu
University of Luxembourg

Abstract: The group $\mathrm{Out}(\mathrm{F}_2)$ of outer-automorphisms of F_2 the rank 2 free group acts naturally on the $PSL(2,\mathbb{C})$-character variety of F_2. To study the dynamical property of $\mathrm{Out}(\mathrm{F}_2)$-action, Bowditch's Q-condition and the primitive stable condition on a representation from F_2 to $PSL(2,\mathbb{C})$ have been introduced by Bowditch (generalized by Tan-Wong-Zhang) and by Minsky, respectively. Each one of them can characterize an open subset of the character variety on which $\mathrm{Out}(\mathrm{F}_2)$ acts properly discontinuously. These two open sets are both candidates for the maximal domain of discontinuity for the $\mathrm{Out}(\mathrm{F}_2)$-action. In a joint work with Jaejeong Lee, we show that these two conditions are equivalent to each other.

Fiber surfaces and Heegaard surfaces of 3-manifolds I & II

Makoto Sakuma
Hiroshima University

Abstract: While fiber surfaces and Heegaard surfaces play a key role in 3-manifold theory, they are quite different in nature. However, we can find various analogies between them. In this talk, we explain the analogies from the view point of

1. branched virtual fibration theorem
2. monodromy groups (joint work with Donghi Lee and that with Ken'chi Ohshika)
3. variantions of McShane's idenitity (joint work with Hirotaka Akiyoshi and Hideki Miyachi and that with Donghi Lee)
4. the space of Kleinian groups (joint work with Hirotaka Akiyoshi and Masaaki Wada and Yasushi Yamashita).

Thursday, January 11

Black holes in general relativity I & II

Makoto Nozawa
Kyoto University

Abstract:

Black holes are one of the most interesting objects in general relativity. It has been known that the stationary black holes in four dimensional vacuum general relativity are exhausted by Kerr family. After reviewing some properties of Kerr black holes, I will discuss how the uniqueness theorem fails to hold in higher dimensions.

Curvature and the second variation of arc length

Jaigyoung Choe
Korea Institute of Advanced Study

Abstract: As an introduction to the second lecture, two theorems will be proved by using the second variation of arc length:
(*Myers' theorem*) Any compact manifold of positive Ricci curvature must have finite fundamental group.
(*Frankel's theorem*) If Σ_1 and Σ_2 are compact immersed minimal hypersurfaces in a complete connected Riemannian manifold of positive Ricci curvature then Σ_1 and Σ_2 must intersect.

Manifolds of Ricci curvature bounded from below

Jaigyoung Choe
Korea Institute of Advanced Study

Abstract: Let M be a compact Riemannian manifold of nonnegative Ricci curvature and Σ a compact embedded 2-sided minimal hypersurface in M. It will be proved that there is a dichotomy: If Σ does not separate M then Σ is totally geodesic and $M \setminus \Sigma$ is isometric to the Riemannian product $\Sigma \times (a, b)$; If Σ separates M then the map $i_* : \pi_1(\Sigma) \to \pi_1(M)$ induced by inclusion is surjective. This theorem has generalizations and applications. (Joint with A. Fraser)

Friday, January 12

3D gravity and Teichmüller theory I & II

Francesco Bonsante
University of Pavia

Abstract: Gravity in dimension 3 reduces to the study of Lorentzian metrics of constant curvature. While the local problem is trivial, the moduli space of Lorentzian metrics on a manifold is in general a non trivial space. The study of this moduli space have attracted some attention from physicists: on one hand it is considered a toy model to define a proper quantization, on the other hand some interesting phenomena, like black holes, seem to have their corresponding counterpart in this simplified version.

In 1990 Mess pointed out some deep and striking connections between 3D gravity and Teichmüller theory. Using Thurston theory he gave a beautiful and concrete description of the moduli space of of globally hyperbolic structures on $\Sigma \times \mathbb{R}$ of constant curvature in terms of the Teichmüller space of Σ. Since then such connections have been deepened and widely exploited.

In the first part of the talk I will briefly introduce 3D gravity and review Mess' ideas, focussing on the negatively curved case (Anti de Sitter case). In the second part of the talk I will describe some more recent developments of the theory, and some applications to Teichmüller theory.