Geometry of Moduli Space of Low Dimensional Manifolds

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Monday, December 12

Free boundary minimal surfaces I & II

Martin Li
Chinese University of Hong Kong

Abstract: The purpose of these two talks is to give a survey of free boundary minimal surfaces in Riemannian three-manifolds with boundary. Part I of the talk will cover elementary notions of free boundary minimal surfaces including their first and second variations. We will describe some classical existence and regularity results. In Part II, we will describe some more recent advances in the field, with particular emphasis on the Euclidean unit ball and also a general existence theory by min-max constructions.

Interrogating length spectra and quantifying isospectral finiteness I & II

Hugo Parlier
University of Fribourg

Abstract: Associated to a closed hyperbolic surface is its length spectrum, the set of the lengths of all of its closed geodesics. Two surfaces are said to be isospectral if they share the same length spectrum. There are different methods to produce surfaces that are isospectral but not isometric, the most successful one based on a technique introduced by Sunada.

The talks will be about the following questions and how they relate.

- How many questions do you need to ask to determine a length spectrum?
- In a given genus how many different surfaces can be isospectral but not isometric?

The approach to these questions will include finding adapted coordinate sets for moduli spaces and exploring McShane type identities.

Tuesday, December 13

Kontsevich's eye and associators I & II

Hidekazu Furusho
Nagoya University

Abstract: In my talk, I will talk about a new associator constructed by Alekseev and Torossian by reviewing a way to compactify certain "configuration spaces" and the definition of Kontsevich's weights of Lie graphs.

Three-holed sphere groups in $\mathsf{PGL}(3, \mathbb{R})$, I

Sungwoon Kim
Jeju National University

Abstract: The notion of Anosov representation is a generalization of convex cocompact representation in hyperbolic 3-space to higher rank symmetric spaces. In this talk we shall introduce this notion by looking at typical examples in the $\mathsf{PGL}(3, \mathbb{R})$ character variety.

For example, hyperconvex representations introduced by Labourie and horocyclic representations introduced by Barbot when the domain group is the surface group, and positive representations introduced by Fock and Goncharov, and Pappus representations introduced by Schwartz when the domain group is the free group (of rank 2).

While discussing these examples, we shall focus on the property of boundary-embeddedness (slightly weaker notion than Anosov) with our motivational questions in mind for the second talk.

(Work in progress joint with Jaejeong Lee)

Three-holed sphere groups in $\mathsf{PGL}(3, \mathbb{R})$, II

Jaejeong Lee
Korea Institute of Adanace Study

Abstract: As shown in the first talk, among Anosov representations in the $\mathsf{PGL}(3, \mathbb{R})$ character variety of the free group, there are representations that are *negative* in a certain sense, as well as the well-known positive (hyper-convex) representations. In this second talk, we focus on such representations which come from the framed representations (in the sense of Fock-Goncharov) of the 3-holed sphere group.

In order to exhibit concrete examples we restrict our attention to the (real 2-dimensional) relative character variety where the boundary monordomies are all *quasi-unipotent*. Boundary-embedded and transversal (or antipodal) representations in this subvariety may be called *relatively Anosov* and all such examples properly include the Pappus representations studied by R. Schwartz in 1993.

We further restrict to a certain real 1-dimensional subvariety consisting of representations with 2-fold symmetry and find all boundary-embedded ones in there, thereby obtaining a $\mathsf{PGL}(3, \mathbb{R})$ analogue of R. Schwartz's solution to the Goldman-Parker conjecture on the ideal triangle reflection groups in $\mathsf{PU}(2, 1)$.

(Work in progress joint with Sungwoon Kim)

Wednesday, December 14

Geometric equivalence among smooth section-germs of vector bundles with respect to structure groups I & II

Shyuichi Izumiya
Hokkaido University

Abstract: We consider equivalence relations among smooth section-germs of vector bundles with respect to structure groups. One of these equivalence relations is a slight generalization of G-equivalence among smooth map germs introduced by Tougeron [6]. Altough Gervais [1, 2, 3] investigated some properties of G-equivalence, there are no proper examples in their papers. We give several interesting applications including quantum chemistry and spintronics [5], which are expected to have an application to the theory of topological insulators and so on. Our equivalence relation is a slight generalization of G-equivalence. However, we emphasize that we can open a new door for some important applications of singularity theory. Moreover we define other equivalence relations with respect to structure groups of vector bundles.

Then we discover that there are a lot of previous applications or new applications of the theory of singularities as examples of these equivalence relations. In this talk I will start to explain the celebrating results of Mahter [4] and its generalizations by Tougeron for non-speciaists.The goal of my talk is an explanation of the moduli space to describe the difference of Mahter's $\mathcal{K}$ or $\mathcal{A}$-equivalence between our geometric equivalence.

References

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Thursday, December 15

On the geometry and topology of initial data sets in General Relativity I & II

Greg Galloway
University of Miami

Abstract: An initial data set in spacetime consists of a spacelike hypersurface V, together with its induced (Riemannian) metric h and its second fundamental form K. After a brief introduction to spacetime geometry and general relativity, we will present some topics of recent interest related to the geometry and topology of initial data sets. In particular, after reviewing Hawking's theorem on black hole topology, we will consider the topology of black holes in higher dimensional gravity, inspired by certain developments in string theory and issues related to black hole uniqueness.

We shall also discuss recent work on the geometry and topology of the region of space exterior to all black holes, which is closely connected to the notion of topological censorship. Many of the results to be discussed rely on the recently developed theory of marginally outer trapped surfaces, which are natural spacetime analogues of minimal surfaces in Riemannian geometry.

These talks are partially based on work with a number of collaborators: Lars Andersson, Mattias Dahl, Michael Eichmair, Dan Pollack and Rick Schoen.

Inverse scattering on non-compact manifolds with general metric I & II

Hiroshi Isozaki

University of Tsukuba

Abstract: The problem we address is the inverse scattering on non-compact Riemannian manifolds (or orbifolds, or more generally, manifolds with conical singularities) $\mathcal{M}$ consisting of a union of open sets :

$$\mathcal{M} = \mathcal{K} \cup \mathcal{M}_1 \cup \cdots \cup \mathcal{M}_{N+N'},$$

where $\overline{\mathcal{K}}$ is compact, $\mathcal{M}_i$ is diffeomorphic to $(1, \infty) \times M_i$, M_i being a compact $n-1$ dimensional manifold (or orbifold) endowed with the metric h_{M_i}. On each end $\mathcal{M}_i$, the metric of $\mathcal{M}$ is assumed to behave like

$$ds^2 \sim (dr)^2 + \rho_i(r)^2 h_{M_i}, \quad r \to \infty.$$

Typical examples of $\rho_i(r)$ are

$$\rho_i(r) = A \exp\big(c_0 r + \frac{\beta}{1-\alpha} r^{-\alpha}\big) \quad \text{with} \quad 0 < \alpha < 1, \quad \text{or} \quad A \exp\big(c_0 r\big) r^{\beta}. \qquad (1)$$

We assume that for $1 \leq i \leq N$, the end $\mathcal{M}_i$ has a regular infinitiy, i.e. in (1) $c_0 \geq 0$ and if $c_0 = 0$, then $\beta > 0$, while for $N+1 \leq i \leq N+N'$, it has a cusp, i.e. $c_0 \leq 0$ and if $c_0 = 0$, then $\beta < 0$. So, it includes ends such as regular infinities of hyperbolic manifolds : $\rho(r) = e^r$, euclidean ends : $\rho(r) = r$, cusps of hyperbolic manifolds : $\rho(r) = e^{-r}$.

By observing the asymptotic behavior at infinity of generalized eigenfunctions for the Laplace operator on $\mathcal{M}$, we introduce the S-matrix (phyiscal S-matrix for regular ends and generalized S-matrix for cusps), and then solve the inverse scattering problem, i.e. the recovery of the manifold $\mathcal{M}$ from one component of the S-matrix (for all energies).

This is a joint work with Y. Kurylev and M. Lassas.

Friday, December 16

Neck-Pinching of $\mathbb{C}P^1$-structures.

Shinpei Baba
University of Heidelberg

Abstract: We discuss about a certain geometric structure (locally homogeneous structure) on a surface and its degeneration. A $\mathbb{C}P^1$-structure on a surface is an atlas modeled on $\mathbb{C}P^1$ with transition maps in $PSL(2, \mathbb{C})$, and it can be regarded as a Riemann surface with a holomorphic quadratic differential. Moreover each $\mathbb{C}P^1$-structure has a holonomy representation from the fundamental group of the surface into $PSL(2, \mathbb{C})$.

Given a diverging one-parameter family of $\mathbb{C}P^1$-structures such that its holonomy representation converges, we would like to characterize it divergence. We discuss about its limit in the special case when its conformal structure is "pinched" along a loop.

Rational represerentation of mapping class group acting on Teichmüller space I & II

Toshihiro Nakanishi
Shimane University

Abstract:

We show that a tuple of $6g-5$ geodesic length functions defined on the Teichmüller space of genus $g \geq 2$ gives a coordinate-system of the Teichmüller space and the action of the mapping class group on it can be represented by a group of rational transformations in this coordinate-system. The rational representation has several applications. One of them is a list of presentations by Dehn twists of all finite subgroups of the mapping class group of genus 2 up to conjugacy.

This is a jint work with Gou Nakamura.