1 Scattering in networks

The *switching phenomenon* in a simplest network constructed as a Quantum Dot (Domain) and few semi-infinite wires appears as a special dependence of transmission coefficients for the Schrdinger equation on the network from the potential on the domain. In case when the domain Ω_0 has a smooth boundary and the wires are "not too wide", $\frac{\delta_0}{\operatorname{diam}\Omega_0} < 1/4$, the transmission coefficients from the input wire Ω_1 to the each of output wires Ω_s is defined, on the resonance energy, by the "shape" of the resonance eigenfunction Ψ_0 of the Schredinger operator $-\triangle\Psi_0 + \mathcal{E}\langle x, \nu\rangle\Psi_0 = \lambda_0\Psi_0$ on the domain. For instance, for Dirichlet boundary conditions we have near resonance eigenvalue λ_0 the following expression for transmission coefficients:

$$\mathcal{S}_{1s}(\lambda) \approx -\frac{2i\sqrt{\lambda_0 - \frac{\pi^2}{\delta^2}}}{\frac{\sum_{t=1}^{4}|\langle\frac{\partial\Psi_0}{\partial n_{\delta_t}}, e_t^1\rangle|^2}{\lambda_0 - \lambda} + i\sqrt{\lambda_0 - \frac{\pi^2}{\delta^2}}} \frac{\langle\frac{\partial\Psi_0}{\partial n_{\delta_1}}, e_1^1\rangle\langle\frac{\partial\Psi_0}{\partial n_{\delta_s}}, e_s^1\rangle}{\sum_{t=1}^{4}|\langle\frac{\partial\Psi_0}{\partial n_{\delta_t}}, e_t^1\rangle|^2} \tag{1}$$

Manipulation of transmission coefficients is executed via rotation of the unit vector ν in the plane parallel to the plane of the device. One may found a triadic quantum switch on this switching phenomenon.

2 Quantum approach to the Turing Stopping Problem

We suggest a solvable mathematical model of an imaginable Stopping Program Testing Device which may, in principle, distinguish if the program run by the computer may stop, or runs indefinitely.

The imaginable device is presented in form of a Quantum Dot with a Quantum Wire attached to it . The corresponding Scattering Matrix $\mathcal{S}_\beta : E \to E$, $\dim E = \infty$, has a form

$$\mathcal{S}_\beta(k) = P_\beta^\perp + \frac{ik - \beta\mathcal{M}^{-1}\beta^+}{ik - \beta\mathcal{M}^{-1}\beta^+}P_\beta, \tag{2}$$

where P_β is an orthogonal projection onto the one-dimensional subspace spanned by the vector β which defines the connection between the Quantum Dot and the wire, M is the Weyl-Titchmarsh function of the Quantum Dot.

Non-clicking of the device may be a signal that the program either will never stop,$\mathcal{S} = I$, or, probably the testing vector has been chosen from an "indistinguishable" set $\mathcal{F}_\varepsilon = \{x : \langle\mathcal{S}x, x\rangle \ \geq |x|^2 - \varepsilon|x|_1^2\}$, which may be described as an ε-cone of elements in E which are "almost orthogonal" to the vector β. We assume that the role of test-vectors in E is played by trajectories of a random walk with non-uniform time-steps Δ_l, introduce proper dot product $\langle x, y\rangle_1$ and the corresponding Wiener measure $\tilde{W}$ on the entrance-space E

and impose proper condition on β :

$$|b|^2_{l_2} = \sum_{m=1}^{\infty} m^2 |\beta_m|^2 := |b|^2 < \infty. \tag{3}$$

Theorem 2.1 *If the above condition (3) is fulfilled then the probability $\tilde{W}(\mathcal{F}_{\varepsilon,1})$ of the Indistinguishable set $\mathcal{F}_\varepsilon$ is small and may be estimated as*

$$\tilde{W}(\mathcal{F}_{\varepsilon,1}) < \frac{\sqrt{\epsilon}}{\prod_l \Delta_l \sqrt{\epsilon + |b|^2}}. \tag{4}$$

The a-posteriori probability of stopping after a series of N independent experiments with "non-click" results may be estimated via Bayes formula .