Nonlinear Schrödinger equations with double well potential

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Gakushuin University Tokyo, January 16, 2010

Abstract: We consider a class of Schrodinger equations with a symmetric double-well potential and a nonlinear perturbation with nonlinearity power μ . In the semiclassical limit we show that the finite dimensional eigenspace associated to the lowest eigenvalues of the linear operator is almost invariant for times of the order of the beating period associate to the linear operator and the dominant term of the wavefunction is given by means of the solutions of a finite dimensional dynamical system.

In such a finite-mode approximation, the stationary states may bifurcate as the strength of the nonlinear term increases and we observe two different pictures depending on the value μ of the nonlinearity power: a supercritical pitch-fork bifurcation, and a subcritical pitch-fork bifurcation with two asymmetric branches occuring as result of saddle-node bifurcations. The first kind of bifurcation always occurs when $\mu < \mu_{threshold}$, where $\mu_{threshold} = \frac{1}{2} \left[3 + \sqrt{13}\right]$ is a given critical value; in contrast, when $\mu > \mu_{threshold}$ then we always observe the second scenario. The remarkable fact is that such a critical value $\mu_{theshold}$ is an universal constant in the sense that it does not depend on the shape of the double well potential and on the dimension n.

We close the lecture discussing some open problems.

This lecture is based on joint papers with D. Bambusi, V. Grecchi and A. Martinez:

- Grecchi V, Martinez A, Sacchetti A, Destruction of the beating effect for a non-linear Schroedinger equation, Comm. Math. Phys. 227 (2002), 191-209.
- Sacchetti A, Nonlinear time-dependent one-dimensional Schrodinger equation with double-well potential, SIAM J. Math. Anal. 35 (2003), 1160-1176.
- Sacchetti A., Nonlinear double-well Schrodinger equations in the semiclassical limit, J. Stat. Phys. 119, (2005) 1347-1382.
- Bambusi D., Sacchetti A., Exponential times in the one-dimensional grosspitaevskii equation with multiple well potential, Comm. Math. Phys. 275, (2007), 1-36.
- Sacchetti A., Universal critical power for nonlinear Schrodinger equations with symmetric double well potential, Phys. Rev. Lett. 103 (2009), 194101:1-4.