B. Valkó (UW-Madison): Beta-ensembles, carousels and stochastic operators I-III
(Joint with B. Virag (Toronto/Budapest))

The Gaussian β-ensemble provides a generalization of the joint eigenvalue distribution of the Gaussian invariant ensembles (GOE, GUE, GSE). Similar β-generalizations exist for other classical random matrix ensembles (Wishart, Manova, Haar unitary) as well.

In the recent years the various point process limits of these ensembles have been derived and characterized. The characterizations are different from the scaling limits of the classical random matrix models: instead of describing the processes via their joint intensity functions, they are either characterized via their counting function using coupled SDE systems, or as the spectrum of certain random differential operators. For the bulk limit process there is also a geometric characterization using the so-called Brownian carousel, as a certain functional of a hyperbolic Brownian motion.

In these lectures I will describe the known scaling limit results, reviewing some of the existing methods. I will discuss various properties of the limiting operators which can be described as Sturm-Liouville or Dirac type operators with random coefficients. I will also show how to extend the Brownian carousel description in other settings and how some of the limiting operators (and point processes) can be obtained as functionals of certain random paths in the hyperbolic plane. Finally, I will discuss various properties of the bulk limit process: e.g. CLT for the number of points in a large interval, the asymptotic probability of a large gap and a large deviation principle for the asymptotic density.

B. Valkó (UW-Madison): Matrix Dufresne identities
(Joint with B. Rider (Temple))

In 1990 Dufresne showed that if b_t is a Brownian motion and $\mu > 0$ then the integral $\int_0^\infty e^{2(b_t - \mu t)} dt$ is distributed as $\frac{1}{2\xi}$ where ξ has Gamma(μ) distribution. This result provides a starting point for a vast collection of distributional identities for integrated geometric Brownian motion, much of which was pioneered by the work of Matsumoto and Yor. They extended the original Dufresne identity in several settings, studying the finite time integral $\int_0^t e^{2(b_s - \mu s)} ds$ and various related processes. Building on their work later O'Connell and Yor proved a Burke-type property for a system of Brownian queues which was a crucial tool in the recent proof of the integrability of the so-called Brownian directed polymer.

Motivated by a problem in random matrix theory Ramírez and Rider conjectured a Dufresne type identity for matrix processes. The role of the geometric Brownian motion is now played by the solution of the matrix valued SDE $dM = M(dB + (1/2 + \mu)dt)$, $M_0 = I$ where B is a matrix valued Brownian motion. The integral in question is $\int_0^\infty M_t M_t^T dt$ and the resulting distribution was conjectured to be inverse Wishart (a natural matrix extension of the inverse gamma distribution). I will discuss the proof of this conjecture, and various further results that can be considered as matrix extensions of the Matsumoto-Yor theory.