Inverse and direct bifurcation problems

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We consider the nonlinear Sturm-Liouville problem

$$-u''(t) + f(u(t)) = \lambda u(t), \quad t \in I := (0,1), \tag{0.1}$$

$$u(t) > 0, \quad t \in I, \tag{0.2}$$

$$u(0) = u(1) = 0, \tag{0.3}$$

where $\lambda > 0$ is a positive parameter. We assume that $f(u)$ satisfies the following conditions (A.1)–(A.2).

(A.1) $f(u)$ is C^1 for $u \geq 0$ satisfying $f(u) > 0$ for $u > 0$, $f(0) = f'(0) = 0$.

(A.2) $f(u)/u$ is strictly increasing for $u \geq 0$ and $f(u)/u \to \infty$ as $u \to \infty$.

In this talk, we mainly consider inverse bifurcation problems. Namely, from the asymptotic behavior of the bifurcation diagram of (0.1)–(0.3), we determine *the unknown nonlinear term $f(u)$ satisfying the conditions (A.1) and (A.2).* Furthermore, as a direct problem, we establish the precise asymptotic formulas for L^q-bifurcation diagrams for some *given* $f(u)$.